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Corrected Hawking Tunneling Radiation in the Higher Dimensional Reissner-Nordström Black Hole

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Abstract Based on the generalized uncertainty principle, in which the quantum gravitational effects are properly taken in to account, the corrected Bekenstein-Hawking entropy of the higher dimensional Reissner-Nordström black hole, up to the square order of Planck length, has been calculated. Using the corrected entropy, the black hole radiation probability has been calculated in the tunneling formalism, which is corrected up to the same order of the Planck length and a generalized quantum tunneling through the event horizon of the black hole is obtained.

Keywords Hawking radiation · Higher dimensional Reissner-Nordstrom black hole · Tunneling formalism

1 Introduction

Since the original analysis of black hole radiation was done [1, 2], several derivations of Hawking radiation were subsequently presented in the literature [3–5]. None of them, however, corresponds directly to one of the heuristic pictures that visualizes the source of radiation as tunneling. In this method [6–12], the particles are allowed to follow the classically forbidden trajectories, by starting just behind the horizon onward to infinity. The particles then travel back in time, since the horizon is locally to the future of the external region. Thus the classical one particle action becomes complex and so the tunneling amplitude is governed by the imaginary part of this action for the outgoing particle. However, the action for the ingoing particle must be real, since classically a particle can fall behind the horizon. This is an important point of calculations of black hole tunneling. The essence of tunneling based calculations is, thus, the computation of the imaginary part of the action for the process of s-wave emission across the horizon, which in turn is related to the Boltzmann factor for the emission at the Hawking temperature. There are two different methods to calculate the imaginary part of the action: one is by Parikh-Wilczek [6–8] radial null geodesic method

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and another is the Hamilton-Jacobi method which was first used by Srinivasan et al. [9–12]. Later, many people [13–16] used the radial null geodesic method to find out the Hawking temperature for different space-time metrics. Recently, tunneling of a Dirac particle through the event horizon was also studied [17, 18]. All of these computations are, however, confined to the semiclassical approximation only. The issue of quantum gravity corrections is generally not discussed. In [19] it is found that the corrections to the temperature and entropy by including the effects of back reaction knowing the modified surface gravity of the black hole due to one loop back reaction for the Schwarzschild case by radial null geodesic method. As an extension, in [20] also applied this method for a noncommutative Schwarzschild metric. Recently, a problem in this approach has been discussed in [21–23] which corresponds to a factor two ambiguity in the original Hawking temperature. From a pure theoretical point of view one can expect that the properties of black holes might also have played an important role in understanding the nature of gravity in higher dimensions. This expectation has triggered the study of black holes in higher dimensional gravity theories as well as in string theory [24, 25]. However the emergence of the TeV-scale gravity provides a motivation [26–28] for the black hole experiments in the future accelerator such as the CERN Large Hadron Collider. Thus, it is important to investigate the effect of the extra dimensions in the various properties of black holes.

In this article we show that the quantum tunneling probability receives new corrections when quantum gravitational effects are properly taken into account, with respect to the Planck scale. We obtain the radiation tunneling of a higher dimensional Reissner-Nordström black hole, using the corrected Bekenstein-Hawking entropy obtained from the generalized uncertainty principle, in which the gravitational effects are taken into account. In the following, we restrict ourselves to uncharged particles radiation from the event horizon of the higher dimensional Reissner-Nordström black hole. Note that if one considers the charged radiated particles, the trajectories are also subject to electromagnetic forces.

2 The Corrected Bekenstein-Hawking Entropy

A natural candidate for charged black holes of higher dimensional is that of Reissner-Nordström d-dimensional solution of Einstein field equation,

$$ds^2 = f(r)c^2dt^2 - f^{-1}(r)dr^2 - r^2 d\Omega_{d-2}, \tag{2.1}$$

where

$$f(r) = 1 - \frac{2\mu}{r^{(d-3)}} + \frac{\theta^2}{r^{2(d-3)}}.$$

The parameter μ is related to mass M of the black hole

$$\mu = \frac{8\pi G_d}{(d-2)\mathcal{A}_{d-2}} M,$$

where G_d is the d-dimensional Newton constant. $\mathcal{A}_{d-2}$ is the area of the unit $(d-2)$-sphere given by

$$\mathcal{A}_{d-2} = \frac{2\pi^{(d-1)/2}}{\Gamma(\frac{d-1}{2})}.$$

The electric charge of the black hole is given by

$$Q^2 = \frac{(d-2)(d-3)}{8\pi G_d}\theta^2.$$

There is an event horizon situated at

$$r_+ = \left[\mu - (\mu^2 - \theta^2)^{\frac{1}{2}}\right]^{\frac{1}{d-3}}, \quad \mu^2 > \theta^2.$$

There is also an outermost horizon situated at

$$r_h = \left[\mu + (\mu^2 - \theta^2)^{\frac{1}{2}}\right]^{\frac{1}{d-3}}, \quad \mu^2 > \theta^2,$$

which has been considered in our previous work [29].

Let us consider the black hole as a d-dimensional cube of size equal to twice its radius r_+, the uncertainty in the position of a Hawking particle, during the emission, is

$$\Delta x = 2r_+ = 2\left[\mu - (\mu^2 - \theta^2)^{\frac{1}{2}}\right]^{\frac{1}{d-3}} = \left[\frac{Q^2 M}{a^2(d-3)}\left(1 - \sqrt{1 - \frac{a^2}{M^2}}\right)\right]^{\frac{1}{d-3}}, \tag{2.2}$$

where $a^2 = \frac{(d-2)}{(d-3)}\frac{Q^2 \mathcal{A}_{d-2}}{8\pi G_d}$. The condition $\mu^2 > \theta^2$ results in $\frac{a^2}{M^2} < 1$.

Using the usual uncertainty principle, uncertainty in the energy of the Hawking particles is

$$\begin{aligned} \Delta E \approx c\Delta p \approx \frac{\hbar c}{\Delta x} &= \hbar c\left[\frac{Q^2 M}{a^2(d-3)}\left(1 - \sqrt{1 - \frac{a^2}{M^2}}\right)\right]^{\frac{-1}{d-3}}, \\ &\approx \hbar c\left(\frac{2(d-3)M}{Q^2}\right)^{\frac{1}{d-3}}. \end{aligned} \tag{2.3}$$

It is easy to obtain the temperature of black hole in d-dimensional space-time. The Hawking temperature is related to the event horizon radius by

$$T = \frac{1}{4\pi r_+} = \frac{1}{2\pi \Delta x}. \tag{2.4}$$

The Bekenstein-Hawking entropy is usually derived from the Hawking temperature. The entropy S may be found from the well known thermodynamics relation,

$$T = \frac{dE}{dS} \approx \frac{dM}{dS}, \tag{2.5}$$

where M means energy and T means temperature. From (2.2), (2.4) and (2.5) we obtain

$$S \approx A_{rea}\left(\frac{2(d-3)}{Q^2}\right)^{\frac{1}{d-3}} \frac{d-3}{d-4} M^{\frac{d-2}{d-3}} + \text{const.}, \tag{2.6}$$

where $A_{rea} = 4\pi r_+^2$ is the surface area of the black hole horizon.

The evaporation of black hole would leave very distinctive imprints on the detectors and temperature of such black hole could be calculated. To study the quantum gravity effects on the Hawking temperature, one can take into account the gravitational effects trough the generalized uncertainty principle. Recently generalized uncertainty principle has been the subject of much interesting works and a lot of papers have appeared in which the usual uncertainty is modified at the framework of microphysics as [30–47]:

$$\Delta x \geq \frac{\hbar}{\Delta p} + L_P^2 \frac{\Delta p}{\hbar}, \tag{2.7}$$

where L_p is the Planck length. The term $L_P^2 \frac{\Delta p}{\hbar}$ in (2.7) shows the gravitational effects to usual uncertainty principle. Let us consider a quantum black hole, an attempt to measure the radius of the black hole, more precisely that is, to make r_+ small thus resulting an increase in Δp, but according to (2.7) for detection of small distances by going to very high momenta, the behavior of the *Heisenberg microscope* changes and a lower bound on the black hole radius r_+ could be obtained. Setting $2r_+$ as Δx_i and inverting (2.7) we obtain

$$\frac{r_+}{L_P^2}\left(1-\sqrt{1-\frac{L_P^2}{r_+^2}}\right) \le \frac{\Delta p}{\hbar} \le \frac{r_+}{L_P^2}\left(1+\sqrt{1-\frac{L_P^2}{r_+^2}}\right). \tag{2.8}$$

From (2.8) one can write

$$\frac{\Delta p}{\hbar} = \frac{r_+}{L_P^2}\left(1-\sqrt{1-\frac{L_P^2}{r_+^2}}\right) = \frac{1}{2r_+} + \mathcal{O}(L_P^4). \tag{2.9}$$

Substitution in (2.7) leads to

$$\Delta x' = \Delta x\left(1+\frac{L_P^2}{4r_+^2}\right). \tag{2.10}$$

Using (2.4), and (2.10) we obtain the Hawking temperature of d-dimensional black hole,

$$T' = \frac{1}{2\pi\,\Delta x}\left(1+\frac{L_P^2}{4r_+^2}\right)^{-1} \simeq T\left(1-\frac{L_P^2}{4r_+^2}\right). \tag{2.11}$$

The corrected entropy S' may be found from the thermodynamics relation (2.5),

$$S' = S\big(1+\alpha L_P^2\big), \tag{2.12}$$

where $S(M)$ is given in (2.6) and

$$\alpha(M) \approx \left(\frac{2(d-3)}{Q^2}\right)^{\frac{1}{d-3}} \frac{d-4}{4(d-2)} M^{\frac{2}{d-3}}. \tag{2.13}$$

Equation (2.12) is the corrected entropy of a higher dimensional charged black hole whose temperature is modified based on the generalized uncertainty principle.

One can show that the radiated energy, through Hawking radiation is the same as that given in (2.3).

3 Black Hole's Radiation via Quantum Tunneling

Classical black holes are perfect absorbers, they accrete their (irreducible) mass and no fraction of it can escape as there are no classical allowed trajectories crossing the horizon on the way out. It is interesting to note how the inclusion of quantum effects allows, for particles in the Reissner-Nordström geometry, to propagate through classically forbidden regions. This suggests that it should be possible to describe the black hole emission process, in a semiclassical fashion, as quantum tunneling. In the WKB approximation the tunneling probability is a function of the imaginary part of the action

$$\Gamma \sim e^{-2\mathcal{I}m(I)}, \tag{3.1}$$

where $\mathcal{I}_m$ is the imaginary part and I is the classical action of trajectory. Equation (3.1) can be written as [29]

$$\Gamma \sim \frac{e^{S_f}}{e^{S_i}} = e^{\Delta S}, \tag{3.2}$$

in which ΔS is the difference between final and initial values of the black hole entropy.

The corrected Bekenstein-Hawking entropy in which the gravitational effects are taken in to account is given by (2.12), so that

$$\Delta S' = \Delta S + \beta L_P^2, \tag{3.3}$$

where

$$\beta = A_{rea}\left(\frac{2(d-3)}{Q^2}\right)^{\frac{1}{d-3}} \frac{d-3}{4(d-2)} M^{\frac{d}{d-3}} \times \left(-1 + \left[1 - \hbar c\left(\frac{2(d-3)}{Q^2}\right)^{\frac{1}{d-3}} M^{-\frac{d-4}{d-3}}\right]^{\frac{d}{d-3}}\right).$$

Substituting (3.3) in (3.2) we obtained

$$\Gamma' \sim \Gamma e^{\beta L_P^2}, \tag{3.4}$$

which shows the corrected tunneling probability up to the square order of Planck length. Appearance of an exponential coefficient in the corrected tunneling probability in (3.4) shows a generalized quantum tunneling through the event horizon of the Reissner-Nordström black hole, which obtains from the quantum gravitational effects on the black hole radiation.

4 Conclusion

Through the generalized uncertainty principle, in which the gravitational effects up to the square order of the Planck length are taken in to account, we were able to calculate the corrected Bekenstein-Hawking entropy of Reissner-Nordström black hole in higher dimensional space-times. Using this corrected Beckenstein-Hawking entropy, we have calculated the quantum tunneling probability of the black hole's radiation, which contains a correction up to the same order in the Planck length. The mathematical consequence of these calculations is a generalized quantum tunneling through the event horizon of the black hole, which comes from the quantum gravitational consideration in the generalized uncertainty principle.

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